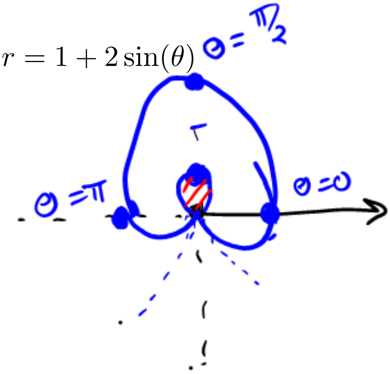
E.g. Find the area of the inner leaf of the curve $\quad r = 1 + 2\sin(\theta)$



find start & stop θ

when $\quad r = 0$

when $\quad 1 + 2\sin\theta = 0$

$$2\sin\theta = -1$$

$$\sin\theta = \frac{-1}{2}$$

know $\quad \sin(\pi/6) = \frac{1}{2}$

} sin is neg

pick: θ start $= 7\pi/6$

θ stop $= 11\pi/6$

picked $\theta_{start} = 7\pi/6$

$\theta_{stop} = 11\pi/6$

sanity check:

thought about how the inside loop was graphed.

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set up & compute

$$\text{area inside} = \int_{7\pi/6}^{11\pi/6} \frac{1}{2} r^2 \, d\theta$$

$$= \int_{7\pi/6}^{11\pi/6} \frac{1}{2} \underbrace{(1 + 2 \cdot \sin\theta)^2}_{\text{FOIL first}} \, d\theta$$

$$= \int_{7\pi/6}^{11\pi/6} \frac{1}{2}\left(1 + 4\sin\theta + 4\cdot\sin^2\theta\right) d\theta$$

$$= \int_{7\pi/6}^{11\pi/6} \frac{1}{2} + 2\cdot\sin\theta + 2\sin^2\theta \; d\theta$$

want antiderivatives

rewrite first

know

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$= \int_{7\pi/6}^{11\pi/6} \frac{1}{2} + 2\cdot\sin\theta + \cancel{2}\cdot\frac{1-\cos(2\theta)}{\cancel{2}} \, d\theta$$

$$= \left[\frac{1}{2}\theta + 2\cdot(-\cos\theta) + \theta - \frac{\sin(2\theta)}{2}\right]_{7\pi/6}^{11\pi/6}$$

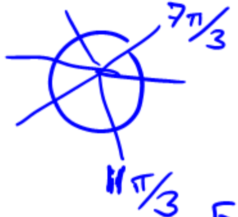
$$\frac{d}{d\theta}\left[\frac{\sin(2\theta)}{2}\right] = \cos(2\theta)\cdot\ldots$$

$$\left[\frac{3}{2}\theta - 2\cdot\cos\theta - \frac{\sin(2\theta)}{2}\right]_{7\pi/6}^{11\pi/6}$$

$$\cos(\pi/6)=\frac{\sqrt{3}}{2}$$

$$= \left(\frac{3}{2}\cdot\frac{11\pi}{6} - 2\cdot\cos\left(\frac{11\pi}{6}\right) - \frac{\sin\left(\overbrace{2\cdot\frac{11\pi}{6}}^{11\pi/3}\right)}{2}\right)$$

$$- \left(\frac{3}{2}\cdot\frac{7\pi}{6} - 2\cdot\cos\left(\frac{7\pi}{6}\right) - \frac{\sin\left(\overbrace{2\cdot\frac{7\pi}{6}}^{7\pi/3}\right)}{2}\right)$$



$$\sin(\pi/3)=\frac{\sqrt{3}}{2}$$

$$= \left(\frac{11}{4}\pi - 2\cdot\left(+\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$- \left(\frac{7\pi}{4} - 2\cdot\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(+\frac{\sqrt{3}}{2}\right)\right)$$

Collecting like terms,

$$= \frac{11\pi}{4} - \frac{7\pi}{4} - \frac{\cancel{2}\sqrt{3}}{\cancel{2}} - \frac{\cancel{2}\sqrt{3}}{\cancel{2}} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{4\pi}{4} - \frac{2\sqrt{3}}{2} - \frac{2\sqrt{3}}{2} + \frac{\cancel{2}\sqrt{3}}{\underset{2}{\cancel{4}}}$$

$$= \boxed{\pi - \frac{3\sqrt{3}}{2}}$$